Washington Township School District

The mission of the Washington Township Public Schools *is to provide a safe educational environment for all students to attain the skills and knowledge specified in the New Jersey Student Learning Standards at all grade levels so as to ensure their full participation in our global society as responsible, self-directed, and civic-minded citizens.*

Course Title:	**Math Explorations and Applications**
Grade Level(s):	**12**

Duration:	*Full Year:*		*Semester:*	**x**	*Marking Period:*	
Course Description:	This course is designed to help students acquire a solid foundation in algebra and geometry, preparing them for other courses such as discrete and finite mathematics in a higher educational setting. Math Explorations helps to show students how algebra and other topics can model and solve authentic real-world problems. It also enables them to develop problem-solving skills, while fostering critical thinking skills. This course is designed for students who have completed Algebra I, Algebra II, and Geometry with an 85-final average or better.					
Grading Procedures:	The board of education policy will be followed as it pertains to secondary grading. Semester Grade The semester grade will be comprised of two marking periods and a final exam. Each marking period will be worth 40%, and the final exam 20%. Marking Period Grade Each marking period will be comprised of tests, quizzes, homework					
Primary Resources:	**The Heart of Mathematics: An Invitation to Effective Thinking 4th Edition** **By:** Edward B. Burger, Michael Starbird					

Washington Township Principles for Effective Teaching and Learning

- Implementing a standards-based curriculum
- Facilitating a learner-centered environment
- Using academic target language and providing comprehensible instruction
- Adapting and using age-appropriate authentic materials
- Providing performance-based assessment experiences
- Infusing 21st century skills for College and Career Readiness in a global society

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Under the Direction of:	Carole English

Written: July 2016

Revised: _____

BOE Approval: _____

Unit Title: Chapter 1. Fun and Games: An Introduction to Rigorous Thought

Unit Description:

Fun and games, rigorous thought: Here we will discover that in mathematics these go hand in hand. Who says that profound ideas and important insights come only from hard work? Sure, we can consider the discipline of mathematics broadly, from a philosophical perspective, and we can be intrigued by its mysterious wonders. But when we get right down to it, we think mathematics is just plain fun.

We start with two fundamental observations:

- Mathematics involves logical and creative thinking.
- Thinking can be fun.

By grappling with conundrums, serious or otherwise, we can discover significant concepts. As we grope for solutions to silly stories, we begin to develop effective strategies for serious thinking.

Unit Duration: 4 days

Desired Results

Standard(s):

Mathematics High School-Modeling

Indicators:

1.1 Silly Stories-Each With a Moral: Conundrums Evoking Thought
1.2 Nudges: Leading Questions and Hints
1.3 The Punch Lines: Solutions and Further Commentary
1.4 From Play to Power: Discovering Strategies of Thought for Life

Understandings:

Students will understand that…

- Interpreting what they read can have a profound effect on how they think.
- Discussing unusual stories and their interpretations with others can help strengthen the effectiveness of their thinking.

Essential Questions:

- Which stories are pertinent to which chapters or ideas?
- What does effective thought look like?
- How can communicating help solve conundrums?

Assessment Evidence

Performance Tasks:

- Divide up the class into groups of 3 or 4 to work out Mindscape Problems.
- Have them record questions, ideas, and potential solutions in their journals.
- Use Mindscapes to help expand their thinking.

Other Evidence:

- Have students record their thoughts and potential solutions in their journals.

Benchmarks:

Completion of Mindscapes from 1.4

Learning Plan

Learning Activities:

- Have students become acquainted while working on some mathematical questions.
- Allow time for students to work with one another and use productive struggle to reach the desired solutions.
- Provide leading questions and hints after a day or two of working with one another.
- Discuss solutions and how they can relate to other mathematic topics.

Resources: The Heart of Mathematics textbook
The WileyPlus online manipulative
Calculators
Student Journals

Unit Learning Goal and Scale

(Level 2.0 reflects a minimal level of proficiency)

Standard(s): Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice.	
4.0	**Students will be able to:** • Develop their own conundrum and modeling solution.
3.0	**Students will be able to:** • identify variables in the situation and select those that represent essential features, formulate a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, • analyze and perform operations on these relationships to draw conclusions, interpret the results of the mathematics in terms of the original situation, • validate the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, • reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle
2.0	**Students will be able to:** • identify variables in the situation and select those that represent essential features, • formulate a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, • analyze and perform operations on these relationships to draw conclusions, • interpret the results of the mathematics in terms of the original situation,
1.0	**With help, partial success at level 2.0 content and level 3.0 content:**
0.0	**Even with help, no success**

Unit Modifications for Special Population Students	
Advanced Learners	Provide writing prompts from the Mindscapes of each unit.
Struggling Learners	Allow use of manipulative to help visualize the mathematical concepts.
English Language Learners	Provide alternate methods of lecture notes.
Learners with an IEP	Each special education student has in Individualized Educational Plan (IEP) that details the specific accommodations, modifications, services, and support needed to level the playing field. This will enable that student to access the curriculum to the greatest extent possible in the least restrictive environment. These include: • Variation of time: adapting the time allotted for learning, task completion, or testing • Variation of input: adapting the way instruction is delivered • Variation of output: adapting how a student can respond to instruction • Variation of size: adapting the number of items the student is expected to complete • Modifying the content, process or product Additional resources are outlined to facilitate appropriate behavior and increase student engagement. The most frequently used modifications and accommodations can be viewed here. Teachers are encouraged to use the Understanding by Design Learning Guidelines (UDL). These guidelines offer a set of concrete suggestions that can be applied to any discipline to ensure that all learners can access and participate in learning opportunities. The framework can be viewed here www.udlguidelines.cast.org
Learners with a 504	Refer to page four in the Parent and Educator Resource Guide to Section 504 to assist in the development of appropriate plans.

Unit Title: Chapter 2. Number Contemplation

Unit Description:

This chapter introduces students to some of the fundamental ideas of number theory and its applications. It explores and develops a sense of number—both integers and reals—as having distinctive characters and features. By the end of the chapter, students should have a sense of a hierarchy of number—natural numbers, integers, rational numbers, and real numbers.

Students should see the significance of intriguing patterns among numbers through their applications to error checking-codes and perhaps cryptography. Students should have a sense of the real numbers and how the rational and irrational numbers sit on the real number line. The hope is students will see numbers as a more varied and interesting body of ideas than they first expected. Beyond the mathematics, the hope is students will begin to learn how to develop new ideas and concepts.

Unit Duration: 36 days

Desired Results

Standard(s):

N-RN B.3, N-Q.A, A-SSE, A-CED, F-BF.A, F-LE

Indicators:

2.1 Counting: Pigeonhole Principle

2.2 Numerical Patterns in Nature: Fibonacci Numbers

2.3 Prime Cuts of Numbers: Prime Numbers

2.4 Crazy Clocks and Check out Bars: Modular Arithmetic

2.5 Secret Codes and How to Become a Spy: RSA Public Key Cryptography

Understandings:

Students will understand that…

- The Pigeonhole Principle is a useful tool for estimation to move from qualitative to quantitative thinking.
- Numerical Patterns exist in nature and contain their own mathematical properties; Fibonacci Sequence.
- Seeking elementary building blocks as a technique for understanding. Building up larger numbers by multiplication evolves into a discussion of factorization.
- The Division Algorithm is presented as an example of clarifying a familiar idea, long division, into a precise statement.
- The Prime Number Theorem concerning the distribution of primes and a discussion of some famous unsolved problems.
- Modular arithmetic and some applications of it to check digits on UPCs and bank checks.
- Looking at things that seem abstract and devoid of application today maybe be central in our daily

Essential Questions:

- How can estimation be used to move from qualitative to quantitative thinking and reasoning?
- How can looking at simple things deeply, finding patterns, and using patterns gain new insights on more complicated situations?
- How can examining the building blocks of complex structures answer old questions, invite new questions to lead to greater understanding?
- How can generalizing a simple idea (like telling time on a clock) lead to important applications?
- How has cryptography evolved over time and how is it applied in every day life?

Assessment Evidence

Performance Tasks:

- Quiz on Sections 2.1-2.2, 2.3, 2.4-2.5
- Test on 2.1-2.3 and 2.1-2.5
- Mindscapes assignments from each section

Other Evidence:

- Research project on Fibonacci
- Research project on Codes and Cryptography
- Student Journal with Mindscape solutions, notes, and summary thoughts.

Benchmarks:

Test 2.1-2.5

Learning Plan

Learning Activities:

- Use a variety of discussion topics, quotes, daily essential questions during lectures.
- Allow time for students to work with one another and use productive struggle to reach the desired solutions.
- Provide leading questions and hints along with appropriate tools to develop deeper understanding of the standards.
- Discuss solutions and how they can relate to other mathematic topics.

Resources: The Heart of Mathematics textbook
The WileyPlus online manipulative
Calculators
Student Journals
Movies: The DaVinci Code, Contact, The Imitation Game

Unit Learning Goal and Scale
(Level 2.0 reflects a minimal level of proficiency)

Standard(s): N-RN B.3

4.0	**Students will be able to:** • Use properties of rational and irrational numbers to explore the Pigeonhole counting principle and develop their own scenario depicting the principle.
3.0	**Students will be able to:** • Use properties of rational and irrational numbers. • Explain why the sum or products of two rational numbers is rational; that the sum of a rational number and irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
2.0	**Students will be able to:** • Use properties of rational and irrational numbers. • Explain why the sum or products of two rational numbers is rational; that the sum of a rational number and irrational number is irrational
1.0	**With help, partial success at level 2.0 content and level 3.0 content:**
0.0	**Even with help, no success**

Standard(s): N-Q.A

4.0	**Students will be able to:** • Use quantitative reasoning to explore to explore the Pigeonhole counting principle and develop their own scenario depicting the principle.
3.0	**Students will be able to:** • Reason quantitatively and use units to solve problems. • Use units as a way to understand problems and to guide the solutions of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. • Define appropriate quantities for the purpose of descriptive modeling. • Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
2.0	**Students will be able to:** • Reason quantitatively and use units to solve problems. • Use units as a way to understand problems and to guide the solutions of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
1.0	**With help, partial success at level 2.0 content and level 3.0 content:**
0.0	**Even with help, no success**

Standard(s): A-SSE

4.0	**Students will be able to:** • Derive and/or explain the formula for the sum of a finite geometric series, and use the formula to solve problems.
3.0	**Students will be able to:** • Interpret expressions that represent quantity in terms of its context. • Use the structure of an expression to identify ways to rewrite it; relate it to the recursive formula in developing the Golden Ratio based on the Fibonacci Numbers. • Write expressions in equivalent forms to solve problems. • Choose and produce equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
2.0	**Students will be able to:** • Use the structure of an expression to identify ways to rewrite it; relate it to the recursive formula in developing the Golden Ratio based on the Fibonacci Numbers. • Write expressions in equivalent forms to solve problems. • Choose and produce equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
1.0	**With help, partial success at level 2.0 content and level 3.0 content:**
0.0	**Even with help, no success**

Standard(s): A-CED	
4.0	**Students will be able to:** • Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
3.0	**Students will be able to:** • Create equations and inequalities in one variable and use them to solve problems. • Include equations arising from linear and quadratic functions, and simple rational and exponential functions. • Create equations in two or more variables to represent relationships between quantities. • Represent constraints by equations or inequalities and interpret solutions as viable or nonviable options in a modeling context.
2.0	**Students will be able to:** • Create equations and inequalities in one variable and use them to solve problems. • Include equations arising from linear and quadratic functions, and simple rational and exponential functions. • Create equations in two or more variables to represent relationships between quantities.
1.0	**With help, partial success at level 2.0 content and level 3.0 content:**
0.0	**Even with help, no success**

Standard(s): F-BF	
4.0	**Students will be able to:** • Find inverse functions. • Verify by composition that one function is the inverse of the other. • Read values of an inverse function from the data. • Produce invertible function from a non-invertible function by restricting the domain. • Use the inverse relationships between other functions.
3.0	**Students will be able to:** • Write a function that describes a relationship between two quantities. • Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
2.0	**Students will be able to:** • Build a function that models a relationship between two quantities. • Build new functions from existing functions.

1.0	**With help, partial success at level 2.0 content and level 3.0 content:**
0.0	**Even with help, no success**

Standard(s): F-LE	
4.0	**Students will be able to:** • Recognize and develop situations in which quantities grow/decay by a constant percent rate per unit interval relative to one another.
3.0	**Students will be able to:** • Distinguish between situations that can be modeled with linear functions and with exponential functions. • Construct functions and arithmetic and geometric sequences given data. • Observe using data that a quantity converges to a specific number or diverges to infinity. • Interpret parameters in a linear or exponential function in terms of a context.
2.0	**Students will be able to:** • Construct and compare linear and exponential models to solve problems. • Interpret expressions for functions in terms of the situation they model.
1.0	**With help, partial success at level 2.0 content and level 3.0 content:**
0.0	**Even with help, no success**

Unit Modifications for Special Population Students	
Advanced Learners	Provide writing prompts from the Mindscapes of each unit.
Struggling Learners	Allow use of manipulative to help visualize the mathematical concepts.
English Language Learners	Provide alternate methods of lecture notes.
Learners with an IEP	Each special education student has in Individualized Educational Plan (IEP) that details the specific accommodations, modifications, services, and support needed to level the playing field. This will enable that student to access the curriculum to the greatest extent possible in the least restrictive environment. These include: • Variation of time: adapting the time allotted for learning, task completion, or testing • Variation of input: adapting the way instruction is delivered • Variation of output: adapting how a student can respond to instruction • Variation of size: adapting the number of items the student is expected to complete • Modifying the content, process or product Additional resources are outlined to facilitate appropriate behavior and increase student engagement. The most frequently used modifications and accommodations can be viewed here. Teachers are encouraged to use the Understanding by Design Learning Guidelines (UDL). These guidelines offer a set of concrete suggestions that can be applied to any discipline to ensure that all learners can access and participate in learning opportunities. The framework can be viewed here www.udlguidelines.cast.org
Learners with a 504	Refer to page four in the Parent and Educator Resource Guide to Section 504 to assist in the development of appropriate plans.

Unit Title: Chapter 4. Geometric Gems

Unit Description:

This chapter presents a spectrum of geometrical insights from the ancient to the modern, from the plane to the fourth dimension. The chapter's overall goal is to help students comprehend the powerful consequences of visualization and geometric relationships. The sections vary as to difficulty and in their proportion of mathematical challenge to pictorial interest.

Unit Duration: 45 days

Desired Results

Standard(s):

G-CO.A.2, 3, G-CO.D, G-GPE.A.1, G-GMD.B

Indicators:

4.1 Pythagoras and his Hypotenuse: Bhaskara's Elegant Proof

4.2 A View of an Art Gallery: View-Obstruction Question from Computational Geometry

4.3 The Sexiest Rectangle: The Golden Rectangle

4.4 Soothing Symmetry and Spinning Pinwheels: Aperiodic Tilings

4.5 The Platonic Solids Turn Amorous: Symmetry and Duality in the Platonic Solids

4.6 The Shape of Reality: Non-Euclidean Geometries

4.7 The Fourth Dimension: Geometry through Analogy

Understandings:

Students will understand that…

- The Pythagorean Theorem presents and states an elegant geometric proof.
- The Art Gallery Theorem can be used to help understand unsolved problems in computational geometry in the plane.
- The Golden Rectangle s constructed with proportions equate to the Golden Mean (from Ch 2).
- The idea of symmetry presents an example and proof of an aperiodic tiling of the plane.
- Regular solids present the idea of duality.
- Investigating these solids can encourage thoughts to move from the qualitative to the quantitative.
- The properties of geometric shapes are altered in spaces other than flat ones.
- Geometry of the four-dimensional Euclidean spaces and the methods of description builds insight/intuition by looking at relationships in lower dimensions.

Essential Questions:

- How can seemingly abstract ideas be made tangible?
- How can manipulating simple shapes lead to profound results?
- How can playing with concrete examples lead to general results?
- How can the interplay between seemingly different disciplines and perspectives (mathematics and art, geometry and nature) use analytics and aesthetics?
- How can making intuitive ideas more precise and lead to greater understanding of completely new creations?
- How can coincidences and patterns reveal underlying structures and relationships?
- How can keeping an open mind to possible realities (even if they are counter to previous experiences) broaden our own world views?
- How can understanding a difficult idea be easier if you think about simpler versions and use analogies to build complexity?

Assessment Evidence

Performance Tasks:

- Quiz on Sections 4.1, 4.2-4.3, 4.4-4.5
- Tests 4.1-4.3 and 4.4-4.6
- Mindscapes assignments from each section

Other Evidence:

- Research project on Pythagorean puzzles and tangrams
- Constructing the 5 Platonic Solids
- Develop an original tiling
- Student Journal with Mindscape solutions, notes, and summary thoughts.

Benchmarks:

Test on Ch 4.1-4.7

Learning Plan

Learning Activities:

- Use a variety of discussion topics, quotes, daily essential questions during lectures.
- Allow time for students to work with one another and use productive struggle to reach the desired solutions.
- Provide leading questions and hints along with appropriate tools to develop deeper understanding of the standards.
- Discuss solutions and how they can relate to other mathematic topics.

Resources: The Heart of Mathematics textbook
The WileyPlus online manipulative
Calculators
Student Journals
Movies: The Thomas Crown Affair, Contact, Donald Duck in math magic land.

Unit Learning Goal and Scale
(Level 2.0 reflects a minimal level of proficiency)

Standard(s): G-CO.A.2, 3

4.0	**Students will be able to:** • Use transformations in the plane to demonstrate rigid motions of geometric figures to create designs, and utilize tangrams to develop new geometric figures.
3.0	**Students will be able to:** • Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. • Compare transformations that preserve distance and angle to those that do not. • Develop definitions of rotations
2.0	**Students will be able to:** • Compare transformations that preserve distance and angle to those that do not. • Develop definitions of rotations
1.0	**With help, partial success at level 2.0 content and level 3.0 content:**
0.0	**Even with help, no success**

Standard(s): G-CO.D

4.0	**Students will be able to:** • Use platonic solids and construct the dual from one solid to another.
3.0	**Students will be able to:** • Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). • Construct Platonic solids from nets, then use solids to develop the Edge-Vertex-Face relationship. • Use platonic solids to investigate the concept of duality.
2.0	**Students will be able to:** • Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). • Construct Platonic solids from nets, then use solids to develop the Edge-Vertex-Face relationship.
1.0	**With help, partial success at level 2.0 content and level 3.0 content:**
0.0	**Even with help, no success**

Standard(s): G-GPE.A

4.0	**Students will be able to:** • Apply ideas of Pythagorean Theorem to then use shapes to develop other 2-dimensional polygons.
3.0	**Students will be able to:** • Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation
2.0	**Students will be able to:** • Translate between the geometric description and the equation of the Pythagorean Theorem.
1.0	**With help, partial success at level 2.0 content and level 3.0 content:**
0.0	**Even with help, no success**

Standard(s): G-GMD.B	
4.0	**Students will be able to:** • Use platonic solids and construct the dual from one solid to another.
3.0	**Students will be able to:** • Visualize relationships between two-dimensional and three-dimensional objects. • Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects
2.0	**Students will be able to:** • Identify shapes of 2-dimensional objects based on cross-sections of 3-dimensional solids.
1.0	**With help, partial success at level 2.0 content and level 3.0 content:**
0.0	**Even with help, no success**

Unit Modifications for Special Population Students	
Advanced Learners	Provide writing prompts from the Mindscapes of each unit.
Struggling Learners	Allow use of manipulative to help visualize the mathematical concepts.
English Language Learners	Provide alternate methods of lecture notes.
Learners with an IEP	Each special education student has in Individualized Educational Plan (IEP) that details the specific accommodations, modifications, services, and support needed to level the playing field. This will enable that student to access the curriculum to the greatest extent possible in the least restrictive environment. These include: • Variation of time: adapting the time allotted for learning, task completion, or testing • Variation of input: adapting the way instruction is delivered • Variation of output: adapting how a student can respond to instruction • Variation of size: adapting the number of items the student is expected to complete • Modifying the content, process or product Additional resources are outlined to facilitate appropriate behavior and increase student engagement. The most frequently used modifications and accommodations can be viewed here. Teachers are encouraged to use the Understanding by Design Learning Guidelines (UDL). These guidelines offer a set of concrete suggestions that can be applied to any

	discipline to ensure that all learners can access and participate in learning opportunities. The framework can be viewed here www.udlguidelines.cast.org
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Interdisciplinary Connections

Indicators:
ELA: ELRST.11-12.1, RST.11-12.3, RST.11-12.4, NJSLSA.W5, WHST.11-12.1
Science: HS ETS1 2
Technology: 8.2.12.E.1

Integration of 21st Century Skills

Indicators: CRP1, CRP2, CRP4, CRP6, CRP7, CRP8, CRP11